STAT

Del

Approved For Release 2009/08/17 : CIA-RDP88-00904R000100100

Third United Nations International Conference on the Peaceful Uses of Atomic Energy

A/CONF.28/P/357
USSR
May 1964
Original: RUSSIAN

Confidential until official release during Conference

EFFECTS OF CROSS-SECTION RESONANCE STRUCTURE ON NEUTRON DIFFUSION AND RESONANCE EFFECTS ON FISSIONABLE NUCLEI

By L.P.Abagyan, N.O.Bazazyantz, I.I.Bondarenko, G.A.Guseynov, A.A.Lukyanov, U M.Mahanov, V.I.Melentyev, M.N.Nikolaev, V.V.Orlov, N.N.Rabotnov, A.P.Souvorov, L.N.Usatchyov, V.V.Filippov.

The importance of taking into account the resonance structure of nuclear cross sections (especially for U^{238}) was clearly understood in early investigations on thermal fission theory [1–6]. I.I.Bondarenko pointed out in 1957 that resonance effects are of importance in the fast reactor physics too. Later it was confirmed by macroscopic experiments and corresponding calculations.

In this report the general results of the resonance effects investigations and methods of its applications to the purposes of reactor calculations are presented. Some of these calculations have been already reported [7–15] and used in the construction of a multigroup constant set. The theoretical and experimental problems of resonance capture in heterogeneous thermal reactors are not discussed.

RESONANCE STRUCTURE OF NUCLEAR CROSS SECTIONS AND REACTOR CALCULATIONS

When considering the energy spectrum of neutrons diffusing in the medii two points could be marked.

First, the neutron flux varies sharply reflecting resonance structure of the cross sections; these variations form the "Microstructure" of the spectrum. Second, the neutron absorption, diffusion and existence of neutron sources result in rather smooth changing of average neutron flux and its deflections from Fermi spectrum. A number of methods could be used for the determination of neutron space-energy distributions in the case of medii with smoothly varying cross sections. As for the calculation of neutron spectrum "microstructure", the exact solution of the problem depends on the detailed information about the cross-section variations in the wide range of neutron energies and thus seems to be impossible in present.

Fortunately, the purpose of calculations usually is to determine the values averaged over the great number of resonance. (Sometimes the exact information about the neutron flux is nevertheless required. But a number of such resonances is not too large and an individual consideration does not make a trouble). For this aim the knowledge of average neutron flux

$\phi(\vec{r}, u, \vec{\Omega})$ is sufficient. The evaluation of the number of events corresponding to any process can be carried out by using the cross-section values averaged with neutron flux as a weight.

$$\bar{\Sigma}_x = [\int \Sigma_x(u)\phi(\vec{r}, u, \vec{\Omega})\, du] / \int_{\Delta u} \phi(\vec{r}, u, \vec{\Omega})\, du \qquad (1)$$

(Index "x" denotes the type of interaction – capture, fission etc.).

These average cross sections are in general dependent on the direction and coordinates and their evaluation requires the information about the dependence of neutron flux $\phi(\vec{r}, u, \vec{\Omega})$, which is in general available. If the diffusion and (or) energy interval used for averaging are so small that

$$|D_\Delta 2\phi/\phi| \ll \Sigma_a + \zeta\Sigma_4/\Delta u \qquad (2)$$

(but Δu is at the same time great enough for average cross sections to be smooth) the flux $\phi(\vec{r}, u, \Omega)$ in the case of one-dimensional geometry has a form

$$\phi(\vec{r}\ u, \vec{\Omega}) = (1/4\pi)\sum_l (2l + \lambda)\,\bar{\phi}_l(u, \vec{r})\ \phi_l(u)p_l(\mu) \qquad (3)$$

Here $\bar{\phi}_l$ is smooth function U and $\phi_l(u) = \phi_l(u, \vec{r})/\bar{\phi}_l(u, \vec{r})$ describes the microstructure of spherical harmonic spectra and depends on the medium's properties only. Notice that although relation (2) is the condition of the applicability of diffusion approximation the results are valid also for higher approximations. The condition (2) usually holds if regions far from the boundaries as well as from the localized neutron sources are under considera-ti... ...e can use available average constants for the description of neutron propagation in ... regions. For more accurate determination of the neutron flux distribution near the boundaries special consideration is needed. It is true also for the problem of neutron flux determination in the case of medii with deep interference minima in total cross section when condition (2) also fails to hold in principal. We restrict ourselves to the region of problems (which is wide enough) where condition (2) holds.

KINETIC EQUATION IN l – REPRESENTATION

The system of equation for moments ϕ_l (x, u) has a form (for simplicity the plane geometry only is considered)

$$\phi_o\Sigma + \partial\phi_1/\partial x = Q(x_1 u) + \sum_i \int \phi_o \Sigma^i_{so}(u^1 \to u)du^1 + \int \phi_o \Sigma_{ino}(u^1 \to u)du^1$$

$$\phi_e\Sigma + \frac{1}{2l+1}\{(l+1)\partial\phi_{eff}/\partial x + l\,\partial\phi_{l-1}/\partial x\} = \sum_i \int \phi_e \Sigma^i_4(u^1 \to u)du^1 + \int \phi_l \Sigma_{inl}(u^1 \to u)du^1 \qquad (4)$$

Here $l = 1, 2, \ldots$, Σ – is the total cross section, Σ^i_{sl}, Σ_{inl} is the i – th component of medium and inelastic cross section. As we supposed the quantity $\partial\phi_1/\partial x$ has a little value and the integrals in (4) as well as quantity Q are the smooth energy functions. That is why we may adopt the next expression for neutron flux: $\phi_o = \Psi/\Sigma(u) \sim 1/\Sigma(u)$

357

$\phi_o = \frac{\Psi}{\Sigma(u)} \sim \frac{1}{E(u)}$ where Ψ – collision density.

In the same way

$$\phi_l(u) = \sum_{n=0}^{l} a_{l-n}/\Sigma^{n+1}(u) \qquad (5)$$

where coefficients a_{l-n} are determined by recursion formulae:

$$a_l = 1/(1 - <\Sigma_s f_l \Sigma / \sum_{n=0}^{l-1} <\Sigma_s f_l / \Sigma^{n+2}> a_{l-1-n+1} \qquad (6)$$

Here $\Sigma_s f_l = \sum_i \int \Sigma_{sl}(u^1 \to u) du$

By averaging Eqs (4) we have

$$\bar{\phi}_0 \bar{\Sigma}_0 + \partial \overline{\phi_1}/\partial x = Q + \sum_i \int \bar{\phi}_0 (x_1 u^1) \bar{\Sigma}^i_{so} (u^1 \to u) du^1 + \int \bar{\phi}_0 \Sigma_{ino} (u^1 \to u) du^1$$

$$\bar{\phi}_l \bar{\Sigma}_l + \frac{1}{2l+1} \{(l+1)(\partial \bar{\phi}_{l+1}/\partial x) + l(\partial \bar{\phi}_{l-1}/\partial x)\} = \sum_i \int \bar{\phi}_l \bar{\Sigma}_{sl} (u^1 \to u) du^1 + \int \bar{\phi}_l \Sigma_{ino} (u^1 \to u) du^1 \qquad (7)$$

where averaged cross sections are determined by the relations

$$\bar{\Sigma}_l = <\phi_l \Sigma>/<\phi_l>, \quad \bar{\Sigma}^i_{se}(u^1 \to u) = <\Sigma^i_s (u^1 \to u) \phi_l(u^1)>/<\phi_l(u^1)>$$

$$\bar{\Sigma}_x = <\Sigma_x/\Sigma>/<1/\Sigma>, \quad \Sigma_{inl}(u^1 \to u) = <\Sigma_{in}(u^1 \to u) \phi_l(u^1)>/<\phi_l(u^1)> \qquad (8)$$

and ϕ_l (u) is determined by Eq. (5).

The discussed averaging technique is based upon the assumptions (in addition to (2)):

1. The collision density variations in averaging interval Δu because of Plachek spectrum oscillations following every resonance are ignored. These oscillations change microstructure of neutron spectrum (in particular $\phi_o(u)$ differs from $1/\Sigma$ (u) which can be taken into account without difficulties by discussed method. But this method is hardly important for the range of neutron energies where statistical description of resonance structure is valid. Really the contribution of every single resonance into the collision density is comparatively small. By taking into consideration the level spacing distribution and rather sharp variation with energy this spacing divided by average energy loss one gets more smooth collision density. This effect may be significant only for a few low lying resonances which require individual treatment.

357

2. Resonances are supposed to be narrow relative to the scattered neutron energy loss. As a rule this condition is invalid only for a few first resonances which are to be considered individually.

3. For the determination of spectrum ϕ_l (u) it had Δu been supposed in Eqs (4)

$$\int \bar{\phi}_l (u^1) \bar{\Sigma}_{sl} (u^1 \to u) du^1 \infty < \phi_l \Sigma_s \int l$$

which could be not true in the case of large energy losses (inelastic scattering and scattering by hydrogen). By using iteration method one could calculate the spectrum ϕ_l (u) more accurately.

GRILLING – HERTZEL APPROXIMATION

There are systems where an anisotropy of neutron flux is appreciable only for high neutron energies. In this energy range the inelastic scattering contributes mainly into moderation. In such a situation the elastic moderation could be determined by using the Grilling-Hertzel approximation leading to the next equations

$$\partial/\partial u \{ (1-\partial\gamma)/\partial u) \Sigma_s + \gamma \Sigma_{oif}] \phi_o - \gamma \Sigma - \partial\phi_1/\partial x + Q \int \phi_o \Sigma_{in} \int_{ino}]] + \phi_o \Sigma_{aif} = Q - \partial\phi/\partial x + \int \phi_o \Sigma_{in} \int_{ino}$$

$$\phi_1 [\Sigma_1 - \Sigma_s \int_1] + 1/3 \{\partial\phi_o/\partial x + 2\partial\phi_2/\partial x\} = \int \phi_1 \Sigma_{in} \int_{in}, -(\partial/\partial u)\phi_1 \quad x_1 \Sigma_s$$

$$\phi_l [\Sigma_l - \Sigma_s \int_l] + \frac{1}{2l+1} \{l + (\partial\phi_{l-1}/\partial x) + (\partial x) + (l+1)(\partial\phi_{l-1}/\partial x\} = \int \phi_l \Sigma_{in} \int_{ine} du^1$$

$$l = 2, 3 \ldots\ldots$$

The elastic moderation in the case of light nuclei can be treated in the same way as the inelastic scattering.

Denote the scattering function $\int(\mu_o, u, u^1 \to u)$ moments as

$$x_{en}(u) = \int (u^1 \to u)^n du^1 \int P_l(\mu, u, u^1 - u) d\bar{\Omega} \qquad (10)$$

Then by using Eqs (8) the coefficients of Eq (9) can be written in the next form

$$\xi = < \sum_i x_{01}^i \Sigma_{si}/\Sigma > / < \Sigma_s/\Sigma >, \quad \gamma = < \sum_i x_{02}^i \Sigma_{si} > / \Sigma > / < \Sigma_s/\Sigma > \qquad \epsilon\xi\xi[$$

$$x_{ll} = < \sum_i x_{ll}^i \Sigma_{si} \phi_i > / < \Sigma_s \phi_l >, \quad \sum_s \int_l = < \phi_l \Sigma_i \Sigma_s^i x_{la}^i > / < \phi_l >$$

Moments x_{en} can be found [II] from the next formulae: for hydrogen $x_n = 1$, $x_{10} = 213$, $x_{11} = 419$, $x_{02} = 2$ for nuclei with $A >> 1$

357

Approved For Release 2009/08/17 : CIA-RDP88-00904R000100100043-3

$$x_{01} = 2/A[1-x_{10}(u)],\ x_{02}(u) = 16/3A^2)[1 - 213x_{10}(u) + 1/2x_{20}(u)]$$
$$x_{11}(u) = (-2/3A)[1 - 3x_{10}(u) + 2x_{20}(u)] \qquad (12)$$

Where x_{10} and x_{20} are the moments of angular distribution which is to be found in neutron scattering experiment.

As the averaged in this way cross sections and neutron flux vary smoothly with energy any finite-difference method such as multigroup one may be used. These techniques are developed for the case of smooth cross sections [17]. We will not consider such methods.

BOLTZMAN KINETIC EQUATION

In addition to the solution of transport problem in the spherical harmonic representation methods of direct numerical solution of Boltzman Kinetic Equation have been developed (S_n – method [27], Vladimirov's method [28]. Consider such an equation for neutron flux averaged over the resonances. For this purpose Eqs (7) may be used. Multiplying Eq. 1 by $P_l(\mu)(2l+1)/4\pi$ and summing all the quantities we have

$$\sum_{l=0}^{\infty}[2l+1)/4\pi]\bar{\Sigma}_l\,\bar{\phi}_l\,P_l(\mu)+\mu(\partial/\partial x)\bar{\phi} = Q+\Sigma_i\int\bar{\phi}(x,u^1,\mu^1)\bar{\Sigma}_{si}(u^1\to u,\ \mu_o)du^1 d\vec{\Omega}^1+\int\phi(x,u^1,\mu)\ \bar{\Sigma}_{in}(u^1 - u,\ \mu_o)\,d\bar{\Omega}^1$$

Where
$$\bar{\Sigma}_{si}(u^1\to u,\mu_o) = \sum_{l=0}^{\infty}[2l+1)/4\pi]\bar{\Sigma}^i_{sl}(u^1\to u)P_l(\mu_o)$$

and Σ_{in} in the same way

The sum in the left side of the latter equation may be transformed in the next manner. Let $\bar{\Sigma}_l \to \Sigma_\infty$ if $l\to\infty$. Then this sum is

$$\sum_{l=0}^{\infty}[(2l+1)/4\pi](\Sigma_l-\Sigma_\infty)\bar{\phi}_l\,P_l(\mu)+\Sigma\sum_{\infty l=0}^{\infty}[(2l+1)/4\pi]\phi_l P_l(\mu)=\int\bar{\phi}(x,\mu^1,\ u\,\Sigma^1(u^\prime,\mu_o)\bar{\Omega}^1+\Sigma_\infty\phi(x,u,\mu) \qquad (14)$$

Where
$$\Sigma^1(u_l\mu_o) = \sum_{l=0}^{\infty}[(2l+\lambda)/4\pi](\bar{\Sigma}_l-\Sigma_\infty)P_l(\mu_o)$$

Thus the effect of the total cross-section resonance structure is shown to be equivalent to some additional neutron elastic scattering described by cross section $[-\Sigma^1(u,\mu_o)]$. The total cross section is to be renormalized simultaneously.

Notice that
$$\Sigma_\infty = \lim_{l\to\infty}\bar{\Sigma}_l = \lim_{l\,\infty\,\infty}/<\phi_l\,\Sigma>/<\phi_l>$$

For example in the case of isotropic elastic scattering by $\phi_l \quad 1/\Sigma^{l+1}$ and
$$\Sigma_\infty = \lim_{l\to\infty}<1/\Sigma^l>/<1/\Sigma^{l+1}> = \Sigma_{min}$$

357

where Σ_{min} – the minimum value of cross section in the interval of averaging.

THE AVERAGING OF MICROSCOPIC CROSS SECTIONS FOR SOME ELEMENTS

When the great number of reactor system calculations is required it is desirable to have the set of averaged cross-sections for elements used usually in reactor construction. Then macroscopic cross sections for any real reactor system can be easily found. Thus the question arises: in what manner the cross sections are to be averaged for giving the correct macroscopic characteristics of any combination of elements with resonance cross-section structure. This problem can be solved exactly if only one of the mixture components shows resonance cross section structure within any interval of the averaging. In this case one could find the correct average macroscopic cross section Eqs (8) if the values $<1/\Sigma^n>$, $<\Sigma_{sl}(u^1 \to u)/\Sigma^n>/<\Sigma^n>$ are found in the next way: $<1/\Sigma^n>^{-1/n} = \sum_i Ni <\sigma^i> n$

$$<\Sigma_{sl}(u^1 \to u)/\Sigma^n>/<1/\Sigma^n> = \sum_i N_i <\sigma^i_{sl}(u^1 \to u)>n \qquad (15)$$

where

$$<\sigma^i>_n = <1/\sigma_i + \sigma)^n>^{-1/n} - \sigma, \quad <\sigma^i_{sl}(u^1 \to u)>_n = <\sigma_{sl}(u^1 \to u)/(\sigma^i + \sigma)^n>/1/<\sigma^i + \sigma)^n> \qquad (16)$$

N_i is nuclear density of "i"-th mixture component, σ – is total cross section of the other mixture components calculated per one nucleus of the considered type.

When there are more then one element in mixture with resonance cross section structure the values averaged macroscopic cross sections depend on the location of resonances in every element. In this situation only approximate representation of averaged mixture cross sections can be found in terms of the averaged cross sections of particular elements. The uncertainty of such a representation would be the least one if when averaging macroscopic cross sections (Eqs 15) the average total cross section for all the other elements calculated per nucleus of considered element is taken as σ. The practice shows in this case the uncertainty is usually less then that of the experimental data.

The tabulation of average cross section of individual elements for any constant "delution" cross section helps σ the calculation of macroscopic characteristics for any mixture of such elements [16].

THE SUBGROUP METHOD

The discussed above method of taking into account the resonance cross section structure based on the assumption that neutron structure remains constant throughout the medium to be considered. Inspite of the approximation is shown to be proved in a number of cases there are such a situation when a neutron spectrum structure as well as averaged cross sections Eqs (9) are dependent on coordinates. Such a situation always takes place if the contribution of neutrons which undergo no collisions (sometimes only some collisions) is not negligible. (For example,

357

the regions near the neutron sources or the medium boundary; the energies corresponding to the deep minima in the total cross section curve). Then the information about the cross section structure which is contained in average cross sections dependence (Eqs (12) on the "delution" cross section becomes unsufficient. For the introduction of more complete information required for aforementioned problems such a called "subgroup method" [14] could be used. Suppose the smooth energy dependence of neutron flux is described by multigroup method. Consider the neutron group corresponding energy interval ΔE.

Let us divide these neutrons into some "subgroups" unifying within any subgroup the neutrons with the total cross section value enclosed in small intervals $\Delta\sigma_k$. Then assume all the neutrons of the considered subgroup having the same – average – total cross section as well as differential scattering cross section and reaction cross section. If the intervals ($\Delta\sigma_k$) are small enough, the assumed total cross section will be rather close to real one. The same situation to some degree will take place for the partial cross sections because of the correlation with total cross section. The propagation of considered group in continuous medium with small gradient can be described by using conventional methods with the help of the constants (Eqs (12)) calculated by averaging within the subgroups.

If the neutron spectrum structure is not constant (and so the subgroup contribution depends on the coordinates), one have to calculate the propagation of every subgroup apart. This requires the additional knowledge of group-to-group transition cross sections because of the elastic scattering, inelastic scattering and fission. As for fission and inelastic scattering the problem does not make any difficulty because the resulting energy changing is much more than level spacing. But in the case of elastic scattering transition one have to take into consideration the relation between the energy loss caused by scattering and level spacing. In the isolated resonances region the transition cross sections could be obtained from the detailed cross section structure. For higher energies the statistical description based on average resonance parameters must be used. When the energy loss caused by scattering exceeds the level spacing the elastic transition probability becomes proportional to the contributions of those subgroups into which the transition takes place.

Neutron transitions between all subgroups of given group complicate calculations considerably. To calculate space-energy distributions of neutrons one needs to solve the systems of equations for subgroups by methods developed in thermalization problems.

But subgroups number is not found to be large. For evaluation of averaged parameters in P_1 – and P_3 – approximations 2 and 3 subgroups correspondingly will do.

CALCULATION OF AVERAGED CROSS-SECTIONS

Cross-sections averaged in different ways were shown above to be necessary for determination of mean group-parameters (15). These values can be directly derived from experimental data on cross-sections dependence on energy in low energy range only were details of cross-section structure are available. The energy regions where cross-sections are low anddifficult to measure contribute mainly to values of these parameters.

The problem arising in this connection is to reconstruct the curve of cross-section energy dependence on the base of available experimental data measured with poor resolution. Some theoretical concepts of cross-section structure are to be applied in this case.

CROSS-SECTIONS STRUCTURE IN THE REGION OF ISOLATED RESONANCES

The exact solution of the problem mentioned is possible only in the region of isolated resonances. Energy dependence of cross-sections in the vicinity of isolated level is given by Breit-Wigner formulae:

$$\sigma = [\sigma_0/(1+x^2)](\cos 2\phi_l - x\sin 2\phi_l) + \sigma_p = \sigma_r + \sigma_p \tag{17a}$$

$$\sigma_x = [\sigma_0/(1+x^2)]\Gamma_x/\Gamma \tag{17b}$$

$$d\sigma_s(\mu) = 2\pi\lambda^2 d\mu \Sigma_l P_L(\mu)[B_L^{nn} + \sum_j (B_L - A - xI_L)/2(2l+1)(1+x^2)] = ½ d\mu \Sigma_L P_L(\mu)(\int_L \sigma p + \int_L \sigma_{sr}) \tag{17c}$$

where $\sigma_0 = 4\pi\lambda^2[2J+1)/2(2l+1)]\Gamma_n/\Gamma$, $x = 2(E - E_0)/\Gamma$

Γ_n and Γ are neutron and total widths of resonance, Γ_x= is reaction width, E_0 – resonance energy, λ – neutron wave-length, ϕ_l potential phase-shift corresponding to spin and parity of level in question; σ_r – "reaction cross-section" determined in formal way. In (17b) B_L^{nn} describes the contribution of potential scattering to L_{th} harmonic of scattering cross-section.

$$B_L^{nn} = \sum_{l^1} (2l^1+1)(2l+1)(l^1 0l0\, L0)^2 \sin\phi_{l^1}\sin\phi_l \cos(\phi_{l^1} - \phi_l) \tag{18a}$$

Coefficients B_L, A_L, I_L are determined by resonance parameters and phase-shifts of potential scattering interferring with resonance:

$$B_L = z^2(lJlJ|jL)(\Gamma_n^2/\Gamma^2)\cos 2(\phi_{l^1} - \phi_l) \tag{18b}$$

$$A_L = 2(2J+1)\sum_{l^1}(2l^1+1)(l^1 0l0|L0)^2(\Gamma_n/\Gamma)\sin\phi_{l^1}\sin(2\phi_l - \phi_{l^1}) \tag{18c}$$

$$I_L = 2(2J+1)\sum_{l^1}(2l^1+1)(l^1 0l0|L0)^2(\Gamma_n/\Gamma)\sin\phi_{l^1}\cdot\cos(2\phi_l - \phi_{l^1}) \tag{18a}$$

$(l^1 0l0\ L0)$, $z(lJlJ|jL)$ are the Clebsh-Gordan and Blatt-Bidenharn coefficients tabulated in ref. [18].

The above formulae are valid in the case when the partial neutron l-wave only contributes to resonance formation. The further assumption is that nuclei do not move.

When thermal movement of nuclei is taken into account cross-sections formulae 17(a,b) will have a form [5,18]:

$$\sigma = \sigma_p + \sigma_0\sqrt{\pi}/4R_e[\omega(\zeta x/2 + i\zeta/2)\exp(-2i\phi_l)] \tag{19a}$$

$$\sigma_x = \sigma_0\sqrt{\pi}/4(\Gamma_x/\Gamma)R_e[\omega(\zeta x/2 + i\zeta/2)] = \sigma_0(\Gamma_x/\Gamma)\Psi(\zeta,x) \tag{19b}$$

where $\zeta = \Gamma/\Delta$, $\Delta = 2\sqrt{kTE_0/A}$ Doppler-width of level; ω – is the probability integral of complex argument, tabulated in details (see ref. 19).

Equation (17) may be used instead of (19) only when $\zeta >> 1$ condition (2) holds. Doppler-broadening effects considerably on resonance's form in opposite case.

Doppler-broadening of resonances is not of major interest for consideration of scattering anisotropy, because in the cases when this effect is appreciable scattering resonances with high orbital moments can not be in general treated as isolated.

357

AVERAGING OF CROSS-SECTIONS IN THE REGION OF RESOLVED ISOLATED RESONANCES

In this energy range cross-sections (Eg 16), averaged over large number of resonances may be expressed in terms of mean resonance parameters and evaluated.

The method for evaluation of average cross sections needed for calculations in diffusion-approximation has been discussed in detail in refs [12] and [15]. We represent here more general expressions valid for calculations of cross-sections needed in higher approximations. Derivation procedure is identical with one applied in references cited and is omitted here.

$$<1/(\sigma + \sigma)^n> = [1/(\sigma_p + \sigma)] \sum_{m=0}^{n} C_n^m \, I_{r,m} \qquad (20a)$$

$$<\sigma_s f_L/(\sigma + \sigma)^n> = \sigma_p f_{Lp} > 1/(\sigma + \sigma)^n > + J_{Ln} \qquad (20b)$$

Here

$$I_{r,m} = > \sigma_r/(\sigma+\sigma)^m \qquad (21a)$$

$$J_{Ln} = > \sigma_{sr} f_{Lr}/(\sigma+\sigma)^n \qquad (21b)$$

Define also

$$I_x = > \sigma_x/(\sigma + \sigma) > \qquad (21c)$$

Substitution of Eq (17) or (19) into (21) results in

$$I_{r,m} = \frac{1}{\Delta u} \sum_k \frac{\Gamma_k}{2E_{ok}} \eta_m(\xi_k, \alpha_k, \phi_k) \alpha_k^m \int_{-\infty}^{+\infty} [\frac{\mathrm{Cos}\, 2\phi_k - x \,\mathrm{Sin}\, 2\phi_k}{x^2 - x\alpha_k \mathrm{Cos} 2\phi_k + \alpha_k \mathrm{Cos}\, 2\phi_k + 1}]^m dx \qquad (22a)$$

$$J_{Ln} = \frac{1}{\Delta u} \sum \frac{\Gamma_k^2}{2E_{ok}\Gamma_{nk}} \alpha_k \int_{-\infty}^{+\infty} \frac{\gamma_L - x\beta^2 L)(1+x^2)dx}{(x^2 - x\alpha_k \,\mathrm{Cos}\, \phi_k + \alpha_k \mathrm{Cos}\phi_k + 1)^n}$$

In particular

$$I_{r,1} = \frac{1}{\Delta u_k} \sum_k \frac{\pi\Gamma_k}{2E_{ok}} \alpha_k \frac{\eta_1(\xi_k, \alpha_k, \phi_k)(\mathrm{Cos}2\phi_k - (\alpha_k/2)\mathrm{Sin}^2\phi_k}{[(1 - \alpha_k \mathrm{Sin}^2\phi)(1+\alpha_k \mathrm{Cos}^2\phi_k)]^{1/2}} \qquad (22b)$$

$$I_{r2} = \frac{1}{\Delta u} \sum_k \frac{\pi\Gamma_k}{4E_{ok}} \alpha^2 \frac{\phi_1(\xi_k, \alpha_k, \phi_k)}{[(1 - \alpha_k \mathrm{Sin}^2\phi_k)(1 + \alpha_k \mathrm{Cos}^2\phi_k)^{3/2}} \qquad (22c)$$

$$I_u = \frac{2\pi^2}{\sigma_p \Delta u} \sum_k \frac{\Gamma_k \lambda_k^2}{E_{ok}} \frac{\gamma_{Lk} - \beta_{Lk}\frac{\alpha_k}{2}\mathrm{Sin}\, 2\phi_k}{\sqrt{(1-\alpha_k \mathrm{Sin}^2\phi_k)(1 + \alpha_k \mathrm{Cos}^2\phi_k)}} \qquad (23a)$$

$$J_{L2} = \frac{\pi^2}{2\sigma_p^2 \Delta u} \sum_k \frac{\Gamma_k \lambda^2}{E_{ok}} \frac{2\gamma_{Lk}(2+\alpha_k \mathrm{Cos}\, 2\phi) - \beta_{Lk} d_{Lk} \mathrm{Sin}2\phi_k(4+3\alpha_k \mathrm{Cos}2\phi_k \frac{\alpha_k^2}{2} \mathrm{Sin}^2\phi_k}{[(1 - \alpha_k \mathrm{Sin}^2\phi_k)(1 + \alpha_k \mathrm{Cos}^2\phi_k)]^{3/2}} \qquad (23b)$$

$$I_x = \frac{1}{\Delta u} \sum_k \frac{\pi\Gamma_{xk}}{2E_{ok}} \alpha_k \frac{\eta_I^1(\xi_k, \alpha_k, \phi_k)}{[(1 - \alpha_k \mathrm{Sin}^2\phi_k)(1 - \alpha_k \mathrm{Cos}^2\phi_k)]^{1/2}} \qquad (24)$$

The sum is taken over all levels "k" in interval of averaging Δu

$$\alpha_k = \sigma_{ok}/\sigma_p + \sigma \quad \gamma_L = \sum_j \frac{A_c + B_L}{2(2I+1)}, \quad \beta_L = \sum_j \frac{I_L}{2(2I+1)} \qquad (24)$$

357

The functions η_1^1 and η_m in absence of Doppler-broadening are equal to unity.

These functions were calculated elsewhere [4, 5, 9] for m = 1, 2 in assumption that there in no interference between potential and resonance scattering (ϕ = 0). In this case $\eta_1^1 = \eta_1$. There are no principal difficulties in calculations of these functions when $\phi \neq 0$.

REGION OF UNRESOLVED ISOLATED RESONANCES

Mean resonance parameters may be used for calculations of the integral characteristics (21) in the energy range where resonances may be treated as isolated but are not resolved experimentally.

The cross-sections at fixed energy are the sums of partial cross-sections over possible combinations of spin and parity:

$$I_{r,m} = \sum_\nu I^\nu_{r,m}, \quad J_{Ln} = \sum_\nu J^\nu_{Ln}, \quad I_x = \sum_\nu I^o_x \tag{25}$$

where ν = J, π. Values $I^\nu_{r,m}$, $J^\nu_{L,m}$, I^ν_x may be expressed as follows:

$$I^\nu_{r,m} = \frac{1}{2}\frac{\Gamma_\nu}{\bar{D}_o}\bar{a}^m_\nu \eta_m\mathrm{I}(\bar{\xi}_o, \bar{a}_\nu, \bar{\phi}_o) f_m(\bar{a}_o\, \bar{\xi}_o, \bar{\phi}_o)\int_{-\infty}^{+\infty}\left[\frac{\mathrm{Cos}2\phi_\nu - x\mathrm{Sin}2\phi_\nu}{x^2 - x\bar{a}_1\mathrm{Sin}2\phi_\nu + \bar{a}_\nu\mathrm{Cos}2\phi_\nu + 1}\right]^m dx \tag{26}$$

$$J^\nu_{Ln} = \frac{2\pi\lambda^2 \cdot \Gamma_\nu}{\bar{D}\sigma^n_p}\xi(\bar{a}_\nu, \bar{\gamma}_{L\nu}, \bar{\beta}_{L\nu}, \phi_\nu)\int_{-\infty}^{+\infty}\frac{(\bar{\gamma}_{L\nu} - x\bar{\beta}_{Lo})(x^2+1)^{n-1}dx}{(x^2 - x\bar{a}_\nu\mathrm{Sin}2\phi_\nu + \bar{a}_\nu\mathrm{Cos}\,2\phi_\nu + 1)^n} \tag{27}$$

$$I^\nu_x = \frac{\pi}{2}\frac{\overline{\Gamma_x}_\nu}{D_\nu}\bar{a}_\nu\frac{\eta 1(\bar{\xi}_\nu, \bar{a}_\nu\phi_\nu)x^1_1(\bar{a}_\nu\, \bar{\xi}_\nu\phi_\nu)}{[(1 - \bar{a}_\nu\mathrm{Sin}^2\phi_\nu)(1 + \bar{a}_\nu\mathrm{Cos}^2\phi_\nu)]^{1/2}} \tag{28}$$

where $$a_\nu = 4\pi\lambda^2[(2J+1)/2(2I+1)]\,\bar{\Gamma}_{n\nu}/\Gamma_\nu(\sigma_p + \bar{\sigma} + \sum_{\nu 1=0}\bar{\sigma}_{p,\nu 1} \tag{29}$$

The sum in latter expression's denominator reflects overlapping of resonances with different spins and parities.

$\bar{D}_o$ – is average level spacing for spin J and parity π. All the parameters are evaluated at the mean energy of interval. Functions x^1, x_m, ξ_{Ls} correspond to the effects of fluctuations of partial width. These fluctuations are described by Porter-Thomas distribution [20] and functions mentioned may be calculated numerically. The results of such calculations for some of these functions for may be found in refs [11, 21]. Another aspects of fluctuations effect on average values of cross-sections will be discussed below.

REGION OF RESONANCES PARTIALY OVERLAPPED BECAUSE OF DOPPLER-BROADENING

The calculations of averaged cross-section in the region of partialy overlapped interfering levels were impossible up to date because of the lack of satisfactory theoretical representation of cross-section structure in this energy range. It is possible to estimate the selfindication effects only when overlapping of resonances begins because of Doppler-broadening ($\Delta \quad \bar{D}$, but $r \ll \bar{D}$).

The method for calculation of averaged cross-sections for this case is developed. In this method averaged cross-sections are represented as difference between real average cross-section and some correction which takes into account sellshielding effects and has a dispertion form:

357

Approved For Release 2009/08/17 : CIA-RDP88-00904R000100100043-3

$$\langle\sigma_x/\sigma\rangle/\langle 1/\sigma\rangle = \bar{\sigma}_x - (1/\sigma)(\overline{\sigma_x\sigma} - \bar{\sigma}_x\bar{\sigma})$$

$$\langle 1/\sigma^n\rangle = 1/\bar{\sigma}_n + 1/2n(n+1)(\overline{\sigma^2} - \bar{\sigma}^2)\,\bar{\sigma}^{-n+2} \tag{30}$$

Supposing Doppler-width to be large and level spacing to be constant we calculate this correction

$$\overline{\sigma_x\sigma} - \overline{\sigma_x\sigma} = \sum_\nu \bar{\sigma}_{x\nu}\bar{\sigma}_{p\nu}[2lxp(-2\pi^2/b_\nu^2) + (b_\nu/\sqrt{2\pi})\,\phi_x^\nu/S_x]$$

$$\overline{\sigma^2} - \bar{\sigma}^2 = \sum_\nu \overline{\sigma^2}[2lxp(-2\pi^2/b_\nu^2) + (b_\nu/\sqrt{2\pi})\,\phi^\nu] \tag{31}$$

where $\nu = J, \pi$, $b_\nu = \bar{D}_\nu/\Delta$, functions ϕ_x^ν, ϕ^ν, S_{oc} [10, 12, 21] reflect the effects of reduced width distribution on the cross-section value.

THE EFFECTS OF RESONANCE-POTENTIAL INTERFERENCE ON THE RESONANCE SELFSHIELDING OF CROSS-SECTIONS

The interference between resonance and potential scattering, results in sharp change of the form of resonance line. Cross-section's maximum is not placed at resonance energy and value of this shift is $E_{max} - E_o = (\Gamma/2)tg\phi_l$, maximum cross-section $\sigma_{max} = \sigma_0 + \sigma_{min}$, where $\sigma_{min} = \sigma_p - \sigma_o Sin^2\phi_l$ is reached at energy $E_{min} = E_o - (\Gamma/2)\,ctg\,\phi_l$.

For even-even nuclei which after absorption of s-neutron form compound-states with the single possible spin-value $J = 1/2$ the minimum of cross-section equals to $\sigma_p = (1 - \frac{\Gamma_n}{\Gamma})$. This value may be very small, when the widths of inelastic processes are small. For example Fe^{56} cross-section in the interference minimum of 29 kev resonance equals approximately 0.2 barn and is almost intirely due to the impurities of other isotopes of Fe.

The existence of this minima is evident by of major importance for determination of diffusion coefficient. But the value of effective resonance absorption integral is subject to the same effect in some cases. (Fig. 1).

But interference between potential and resonance soattering may be especially important for calculation of the dependence of effective resonance integral on temperature. (Table 1).

The same effect takes place for other resonances of U^{238} with high energy and large neutron width.

The effect in question may be considerable in the region of partially overlapped levels too (see Fig. 2). Because of interference the condition $\Gamma > D$ does not necesserily imply that there is no resonance structure of cross-sections. Cross-sections become smooth only when large number of decay channel of compound nucleus are open.

EFFECTS OF NONUNIFORM COLLISION-DENSITY

As was noted above, Plackek oscillation of collision density does not effect strongly on averaged cross-section in the case of narrow resonances. However these fluctuations are to be

taken into account for resonances with widths which are comparable with energy loss on the nuclei of the element considered but are essentially less than energy loss due to the scattering on nuclei of other mixture – components. To find energy dependence of collision density one must solve integral equation.

The temperature dependence of absorption resonance integral (barns) for 663eV – level of U^{238} $\sigma = 0$.

Table I

L	Temperature K°		
ϕ	0	300	900
= 0	0.011	0.012	0.014
≠ 0	0.027	0.024	0.021

Effects of resonance selfshielding on the average neutron scattering characteristics of the oxygen.

Table II

	No selfshielding			Selfshielding coefficients			
	$\overline{\Sigma}_s$	$\frac{\overline{\xi\Sigma_s}}{\overline{\Sigma}_s}$	Medium $\frac{\overline{\mu\Sigma_s}}{\overline{\Sigma}_s}$	$\frac{\overline{(\Sigma_s/\Sigma_{tot})}}{\overline{\Sigma_s}(\overline{1)\Sigma_{tot}}}$	$\frac{\overline{(\zeta\Sigma_s/\Sigma_{tot})}}{\overline{\xi\Sigma_s}(\overline{1/\Sigma_{tot}})}$	$\frac{\overline{(\Sigma_s/\Sigma^2_{tot})}}{\overline{\Sigma_s}(\overline{1/\Sigma^2_{tot}})}$	$\frac{\overline{(\mu\Sigma_s/\Sigma^2_{tot}\Sigma_s}}{\overline{\mu\Sigma_s}(\overline{\Sigma_s/\Sigma^2_{tot}})}$
0.2–0.4	3.80	0.134	−0.14 0	0.975	0.968	0.960	0.910
				0.994	0.987	0.976	0.933
				0.995	0.993	0.992	0.950
0.4–0.8	5.60	0.086	0.23 0	0.774	0.788	0.678	1.100
				0.850	0.832	0.767	1.060
				0.908	0.900	0.837	1.060
0.8–1.4	4.30	0.110	0.08 0	0.900	0.860	0.858	1.075
				0.960	0.935	0.930	1.250
				1.000	0.975	1.000	1.110
1.4–2.5	1.75	0.100	0.12 0	0.720	0.684	0.372	2.120
				0.980	0.980	0.937	0.995
				0.992	0.992	0.950	0.995

$$\Psi(u) = \sum_i \int_{u-\varepsilon_i}^{u} \Psi(u^1)\,[\sum_i{}^i\,(u^1 \to u)/\Sigma(u)]\,du \qquad (32)$$

In some cases effects of nonuniform collision density may lead to considerable change of results of averaging.

As an example we can take the effective resonance absorption integral as a function of dilution for 29keV level of Fe^{56}. It is demonstrated in Fig.1. The change of this function relative to the case of reniform collision density is determined by arising of collision density peak in the region of interference minimum where ratio Σ_C/Σ increases sharply. Another example is energy dependence of collision density near the first resonance of U^{238} at 6,7 eV, calculated in [11) by Monte-Carlo method.

357

EFFECTS OF RESONANCE SELFSHIELDING ON THE AVERAGED PARAMETERS OF SCATTERING ANISOTROPY

To apply Eq. (22) to calculation of averaged values of scattering anisotropy parameters one needs to known potential phase-shifts and spins of resonance levels formed after absorption of the neutrons with high orbital momenta. The poor information on this question is available up to date. Relatively complete and reliable data exist only for first levels of light nuclei. The calculations carried out for these nuclei show that resonance selfshielding can effect considerably on average angular distributions and consequently on the neutron diffusion. Averaged characteristics of scattering cross-section on oxygen are represented as an example in the Table 2.

The averaging procedure was carried out for group integrals presented in [16] on the bases of angular distributions measured by Langsdorf et al [22].

The results of calculations may be applied not only to pure oxigen but also to the oxigen nuclei included in UO_2 and H_2O. The fluctuations of collision-density were neglected.

The selfshielding factors of fission and radiative capture cross-sections of Pu^{239}

Table III

E	Pu^{239}		f_f				f_c			
	i	T^o_k \ σ	10^3	10^2	10	0	10^3	10^2	10	0
21.5–46.5 kev	10	300	1.00	0.99	0.97	0.94	1.00	1.00	0.98	0.96
		900	1.00	1.00	0.98	0.97	1.00	1.00	0.99	0.98
		2100	1.00	1.00	0.99	0.98	1.00	1.00	0.99	0.99
10.0–21.5 kev	11	300	1.00	0.98	0.93	0.88	1.00	0.99	0.95	0.91
		900	1.00	0.99	0.97	0.93	1.00	1.00	0.98	0.95
		2100	1.00	0.99	0.98	0.95	1.00	1.00	0.99	0.97
4.65+10.0 kev	12	300	1.00	0.96	0.87	0.80	1.00	0.96	0.87	0.80
		900	1.00	0.98	0.93	0.87	1.00	0.98	0.94	0.88
		2100	1.00	0.99	0.95	0.91	1.00	0.99	0.96	0.92
2.15–4.65 kev	13	300	0.99	0.92	0.79	0.69	0.99	0.92	0.77	0.67
		900	1.00	0.96	0.87	0.79	1.00	0.96	0.87	0.78
		2100	1.00	0.98	0.91	0.88	1.00	0.98	0.91	0.87
1.00–2.15 kev	14	300	0.98	0.88	0.67	0.56	0.98	0.87	0.62	0.51
		900	0.99	0.92	0.78	0.69	0.99	0.91	0.76	0.66
		2100	1.00	0.95	0.86	0.82	1.00	0.95	0.84	0.79
465–1000 ev	15	300	0.97	0.80	0.53	0.42	0.96	0.76	0.46	0.36
		900	0.98	0.86	0.64	0.53	0.97	0.82	0.58	0.47
		2100	0.99	0.90	0.77	0.71	0.99	0.89	0.72	0.64
215–465 ev	16	300	0.93	0.70	0:41	0.30	0.91	0.62	0.33	0.24
		900	0.96	0.78	0.50	0.38	0.94	0.72	0.42	0.31
		2100	0.98	0.83	0.58	0.52	0.96	0.80	0.53	0.45
100–215 ev	17	300	0.91	0.63	0.34	0.25	0.89	0.58	0.28	0.20
		900	0.94	0.69	0.41	0.32	0.92	0.64	0.34	0.26
		2100	0.97	0.75	0.49	0.39	0.95	0.70	0.41	0.32

E / i	Pu^{239} i	T_K^o \ σ	f_f -10^3	-10^2	10	0	f_c 10^3	10^2	10	0
46.5–100 ev	18	300	0.78	0.50	0.37	0.34	0.75	0.44	0.30	0.27
		900	0.83	0.59	0.45	0.42	0.80	0.52	0.38	0.35

The proper theoretical analysis of resonance effects is necessary for construction of multygroup constant system for fissionable nuclei on the basis of available experimental data.

We consider the effects of fission and neutron widths fluctuations on various average characterizing capture to fission ratio of probabilities. There exist different experimental procedures for determination of such ratio. One can measure a) cross-sections of capture σ_γ and fission σ_f; b) effective number of neutron yield per neutron absorbed $\eta = \dfrac{\nu \sigma_f}{\sigma_f + \sigma_\gamma}$ ($\bar{\nu}$ – the average number of neutrons emitted per fission); c) capture to fission ratio (relative measurements) σ_γ/σ_f. Immediate averaging of these values over large energy interval results in following mean characteristics:

$$1)\ \alpha = \bar{\sigma}_\gamma/\bar{\sigma}_f\,,\quad 2)\ \alpha_\eta = \bar{\nu}/\bar{\eta} - 1\,,\quad \text{Where}\quad \eta = \nu\,\overline{\left(\frac{\sigma_f}{\sigma_\gamma + \sigma_f}\right)}\,,\quad 3)\ \langle\alpha\rangle = \left(\frac{\sigma_\gamma}{\sigma_f}\right)$$

The last ration of this kind is $\bar{\alpha} = \bar{\Gamma}_\gamma / \bar{\Gamma}_f$ To obtain this one it's necessary to analyze cross-sections structure in resonance-region. All these ratios are d i f f e r e n t. Among them α is of greatest importance in reactor calculations. To demonstrate quantitative difference between "alpha's" we consider a system of isolated noninterfering levels of compound-nucleus with the same spin and parity. Statistical fluctuations of fission and neutron widths will be taken into account in the way developed by Bethe [29] Oleksa [31], Lane and Lynn [21], Porter and Thomas [20]. The procedure of averaging over and fluctuations we designate by brackets. Then (supposing not to fluctuate) we get

$$\alpha = \Gamma_\gamma \left\langle \frac{\Gamma_n}{\Gamma_\gamma + \Gamma_f + \Gamma_n} \right\rangle \Big/ \left\langle \frac{\Gamma_f \Gamma_n}{\Gamma_f + \Gamma_\gamma + \Gamma_n} \right\rangle,\quad \alpha = \Gamma_\gamma \left\langle \frac{1}{\Gamma_f} \right\rangle,\quad \alpha_\eta = \Gamma_\gamma \left\langle \frac{1}{\Gamma_f + \Gamma_n} \right\rangle,\quad \bar{\alpha} = \frac{\Gamma_\gamma}{\langle \Gamma_f \rangle}\,,\qquad (33)$$

Using x^2 – distribution functions with the number of degrees of freedom which equals ν for fission width and unity for neutron width one can see that

$$\langle\alpha\rangle > \alpha_\eta > \alpha > \bar{\alpha} \qquad (34)$$

α decreases approaching $\bar{\alpha}$ when energy increases even with Γ_f and Γ_γ being constant – only because of neutron width increasing, before neutron p-were becomes important.

In fact s-neutrons absorbed by nucleus with spin I form compound – states with momenta $J = I + 1/2$ and statistical weights proportional to $2J + I$. Thus cross-sections and widths must be averaged separatly for each level set with corresponding values of average fission and recluced neutron width and in general with respective number of fission channels. Eq (30)

357

are to be modified correspondingly for this case, but (31) continues to hold. Resonance interference and overlapping do not change considerably the area under resonance curve and consequently do not affect on a, i.e. do not break inequality $a > \bar{a}$ which is to hold. Calculations for Pu^{239} taking the existence of two resonance subsets into account were made and confirmed this conclusion (see Fig. 3). Values of parameters used in these calculations are indicated in Fig. 3 caption.

For U^{233} $\bar{a} = 0.15$ and $a = 0.35$ [31]. Calculated values are $a = 0.27$ and 0.34 for $\nu = 2$ and 3 correspondingly. Average fission widths for resonance subsets were taken from ref. [32].

But for U^{235} (34) begins to fail approximately at 7eV and higher: for 30 resonances above this energy $\bar{a} = 1.4$ and $a = 1.2$ [33], [34], [39]. This contradiction can be avoided by supposition that some part of weak and broad resonances with small Γ_n^o and large Γ_f have been missed in experiment. Formulae estimated the consequences of such missing quantitatively were derived in [35]. The explane mentioned and some other anomalies in resonance structure of U^{235} in the energy range under consideration – the existance of background in fission cross-section 30 barn / $\sqrt{E}$, sharp difference between experimental distribution of reduced neutron widths and one – channel x^2 – distribution for small Γ_n^o and relatively small observed value of mean fission width for resonances II – 40 $\Gamma_{f(11-40)} = 29$ mev (for first ten resonances this value $\Gamma_{f(1-10)} = 78$ mev, see Fig. 4). Real $\bar{a}$ calculated with level missing taken into account equals 0.85. This value satisfies (34) and corresponds to a (0.85) = 1.15 at $\nu = 3$. This value of a is more in line with experiment.

If this assumption is true, real value of average level spacing and mean reduced neutron width are approximately 1.5–2 times less than observed values.

Integrals in the expression for $<a>$ diverge at $\nu = 1.2$ (33). That means in fact that with such broad distributions of fission widths very small values $<a>$ have considerable probability and Γ_f – value is determined by chance but exceeds $\bar{a}$, a and a_{η_0} greatly. The integrals are finite for $\gamma > 2$ and $<a>$ $3a$ and $2\ a$ for $a = 3$ and 4 correspondingly.

But interference and overlapping of resonances effect on $<a>$ (namely decrease it) considerably. The approximation of isolated levels is justified in the case of Pu^{239} and for this nucleus really $<a>$ $2.5 \gg \bar{a}$ 0.4 as expected for $\nu = 1$. For U^{233} $<a>$ 0.35 $a > \bar{a} = 0.15$. The interference effects and overlapping are strong in the case of U^{235} and $<a>$ is close to $\bar{a}$ for this nucleus.

The effects discussed above may result in following fenomenon. In relative measurement of σ_γ/σ_f with energy resolution better than level spacing σ_γ/σ_f Γ_γ/Γ_f of corresponding resonance at every energy point and oscillates strongly reflecting cross-section structure. When averaged this value will give $<a>$. Selector resolution falls with encreasing energy and when it becomes worse than level spacing, cross-sections would be automatically averaged both in numerator and denominator and $<a>$ would "turn into a" – the mean value of quantity to be measured decreases rapidly without slightest physical reason. It means that results of σ_γ/σ_f measurement carried out with poor resolution – with energy spread being approximately equal to group interval – (but with good statistics) – are of great interest because they give immediately a for the group under consideration.

In 26–group constant system published in [16] calculations of selfshielding factors for fission and capture f_f, f_c for Pu^{239} were carried out with mean parameters from [36]. Fission width was supposed to be described by three-channel distribution as in [37]. Mean-parameters measured in [33] P^{239} and one channel distribution are evidently more in line with experiment. They have been used for calculations of f_c and f_f with results shown in Table 3. Below IkeV resonances were treated as narrow and overlapping of levels with the same J was neglected. Γ_n and Γ_f fluctuations were taken into account also in [37]. The dependence of fission and capture cross-sections on temperature was determined by using functions calculated in [9].

Selfshielding coefficients in KeV-region where resonances begin to overlap because of Doppler-broadening were calculated by method developed in 10. The Table 3 containing f_f and f_c shows that in this region selfshielding for σ_f is stronger than for σ_γ with resulting increasing of α.

Selfshielding factors were interpolated for 12–14 groups. No new calculation of f_f and f_c were carried out for 18–th group.

Doppler-coefficients for fast oxide reactor with volume 2000 liters were calculated with f_c and f_f from Table 3. Results are very close to ones obtained with coefficients published in [16].

EXPERIMENTAL DETERMINATION OF THE AVERAGED CROSS-SECTIONS

The average cross-section calculations by using one-level formulae are valid only in rather narrow region of neutron energies where the effects of resonance interference and contribution of high angular momentum resonances (for which the average parameters usually are poorly known) can be neglected. As for higher energies the calculation of the quantities depending on the structure of the cross-sections becomes unreliable. Thus the problem of experimental determination of such quantities arises.

Earliear [7, 12, 13] the experimental method of obtaining the parameters of the total cross-section structure had been suggested. By analysing the transmission curve one can obtain such quatities as for example $\langle 1/\Sigma^n \rangle$ (n = 1, 2, 3....). The measurements of the other necessary characteristics for the description of the neutron propagation in medium may be obtained in the same way. The assumption is made that the collision density fluctuations do not affect the results significantly.

METHOD

Collimated neutron beam with energy spectrum $f(E)$ passed through sample of variable thickness. By using the ΔE detector with efficiency $\epsilon(E)$ and holding "the good geometry" conditions one was allowed to measured the transmission function T(t):

$$T(t) = N(t)/N(O) = \int_{\Delta E} f(E)\,\epsilon(E)\,l^{-N\sigma(E)t}\,dE \Big/ \int_{\Delta E} f(E)\,\epsilon(E)\,dE \qquad (35)$$

Here N – nuclear density of the sample.

357

Let us introduce $P(\sigma)$ – the total cross-section probability distribution. Then transmission $T(t)$.

$$T(t) = \int \epsilon[E(\sigma)]P(\sigma)l^{-N\sigma t}\, d\sigma / \int \epsilon[E(\sigma)]P(\sigma)d\sigma \tag{36}$$

Here $E(\sigma)$ is the function inverse to $\sigma(E)$; integration expances through the all values of σ. If the energy interval ΔE used in transmission experiment is comparatively small one could choose the detector with nearly constant efficiency in this interval. The very interesting case is one of the detector sensitive to only one partial process (or one combination of partial processes) only, taking place in thin sample to be explored. Then the energy sensitivity of the detector is proportional to the cross-section of the chosen process and thus correlates with the variations of the total cross-section.

Suppose we have the set of such detectors including the usual detector with flat efficiency. The transmission curves measured by such detectors are the next

$$T(t) = \int P(\sigma) l^{-NQt} d\sigma \tag{37}$$

$$T_x(t) = \int \sigma_x[E(\sigma)] P(\sigma) l^{-N\sigma t} \tag{38}$$

$$T_{sL}(t) = \int \sigma_s[E(\sigma)] \int_L [E(\sigma)]P(\sigma) l^{-N\sigma t} d\sigma \tag{39}$$

Now we make a representation of the cross section $P(\sigma)$ probability distribution $P(\sigma)$ as a sum of the δ – functions:

$$P(\sigma) = \sum_{i=1}^{m} a_i \delta(\sigma - \bar{\sigma}_i) \tag{40}$$

This is equivalent to the sunstitution of the real cross-section curve by the histogram in the energy interval ΔE.

Then Eqs (37) – (39) become

$$T(t) = \sum_{i=1}^{m} a_i l^{-N\bar{\sigma}_i t} \tag{41}$$

$$T_c(t) = \sum_{i=1}^{m} b_i l^{-N\bar{\sigma}_i t} \tag{42}$$

$$T_{sL}(t) = \sum_{i=1}^{m} C_i l^{-N\bar{\sigma}_i t} \tag{43}$$

where $b_1 = \sigma_i \bar{\sigma}_{ci}/\bar{\sigma}_c$ $c_i = a_i \dfrac{\overline{(\sigma_s f_e)}}{(\sigma_s f_i)}$, σ_x and $\overline{(\sigma_s f_L)}$ are the reaction (fission, capture etc) cross-section and n th harmonic of the elastic cross-section averaged over the neutron beam spectrum, σ_x and $(\sigma_s f_L)$ – the same quantities averaged over the spectrum of those part of beam neutrons for which average total cross-section has a value σ_s. It is easy to notice that the abovementioned quantities are just the same ones which are presented in subgroup method. The set of transmission curves is necessary for this method. By the simultaneous least squares treatment of these curves one could obtain representation in the form (Eqs (4)). In the same way itis possible to evaluate for any curve the maximum number of the exponents in the curve expansion (justified by experimental accuracy) as well as uncertainty matrixe.

Average cross-sections could be measured with the same detectors by conventional methods. The values of parameters Eq.15 relate to subgroup characteristics in the way:

$$\langle 1/(\sigma + \sigma)^n \rangle = \sum_{i=1}^{m} a_i [1/(\sigma_i + \sigma)]^n \tag{44}$$

357

$$<\sigma_x/(\sigma+\sigma)> = \sum_{i91}^{m} b_i[1/(\sigma_i+\sigma)] \quad (45)$$

$$<\sigma_s f_L/(\sigma+\sigma)^n> = \sum_{i=1}^{m} C_i[1/(\sigma_i+\sigma)]^n \quad (46)$$

Also these values can de obtained just from the transmission curves by using such relations as for instance.

$$<\sigma_s f_L/(\sigma+\sigma)^n> = (N^n/n!)\,\overline{\sigma_s} f_L \int_0^\infty T_{sL}(t) I^{-N\sigma t} t^{n-1} dt \quad (47)$$

which are useful for evaluation of the maximum Power $(<1/(\sigma+\sigma)>^n)$ corresponding to reasonable accuracy.

THE MEASUREMENTS OF THE TOTAL CROSS-SECTION STRUCTURE CHARACTERISTICS

The discussed method depends on availability of the neutron sources with sufficient intensity as well as rather efficient detectors. Therefore until it had been used mostly for studying of the total cross-section structure. Transmission experiments were carried out at the Van-Graaf accelerator by using T (p, n) reaction as a neutron source. Neutrons emitted in backward and forward direction had the energies varying from 10keV up to 3Mev. Measurements of the total cross section characteristics in eV region for heavy nuclei are under development. Neutrons of pulsed fast reactor (LBR) [23] are used by the time-of-flight technique. Banks of BF^3 proportional counters (enriched to 96% B^{10}) were used as a detector.

In addition to earlier reported [14] data the new results are presented in Tables (2). The time-of-flight results relates to energy intervals used in [16].

MEASUREMENT OF THE EFFECT OF RESONANCE SELFSHIELDING ON ANGULAR DISTRIBUTION OF NEUTRONS SCATTERED BY Fe

This measurement is yet the only one relating to partial cross-section self-indication carried out by using aforementioned method. Fe was choosen because of the most prominent resonance structure of the total cross-section in Mev region. On the other hand this element has a great practical interest in connection with its wide utilization as a constructive and shielding material. Fast neutron beam of fast BR-5 reactor after scattering by thin Fe sample was detected by Th^{232} fission chamber with effective threshold nearly 1.5 Mev. The spectrum of detected neutrons was very close to the corresponding part of fission spectrum. Transmission function as well as angular distributions of neutrons scattered by Fe sample after transmission through different thicknesses were measured. By the compatible analysis of transmission curves received in this way the detectable neutrons were divided into two subgroups with the next characteristics.

The results of Table III can not be explained by smooth cross-section variations only. Although this effect undoubtedly takes place the subgroup characteristic difference is mainly caused by resonance cross-section structure. One could notice that anysotropy of neutrons elastic scattering for energies 3–4 Mev caused mostly by resonance scattering.

The results allow to demonstrate the importance of the exact account of the resonance self shielding when the problem of neutron propogation through the thick layers is to be solved. For example the asymptotic relaxation length corresponding to E_n = 1.5Mev is equal to 7.95 cm when calculated in P_4-approximation [26]. The same quantity determined by subgroup method is found to 8.40 cm.

LIST OF REFERENCES

1. *I.P.Gurevich*, Pomeranchouck I.Ia. P/649 Proceedings of Second UN Conference on PUAE. Geneva. 1958.
2. *E.Wigner, E.Creutz, H.Jupnik, I.Snyder*, J.Appl. Phys. 26, 92 and 260 (1955).
3. *A.P.Rudik*, in "Theory of nuclear reactors on thermal neutrons " by *A.D.Galanin* 2-nd ed. M.Gosatomisdat, *1961*.
4. *V.V.Orlov, T.V.Golashvily, A.I.Vaskin* in "Neutron physics". M. Gosatomisdat, 1961, p. 116.
5. *L.Dresner* "Resonance Absorption in Nuclear Reactors". Pergamon Press. 1960.
6. *A.Vecuberg, E.Wigner* "The Physical Theory of Neutron Chain Reactors". University of Chicago Press. 2-nd impression. 1959.
7. *I.Leipunski* et. al. P/2038. Proceedings of Second UN Conference on PUAE. Geneva. 1958.
8. *M.N.Nicolaev, V.V.Filipov, I.I.Bondarenko*, Report N85 in "Physics of Fast and Intermediate reactors" IAEA. Vienna, 1962.
9. *I.V.Gordeev, V.V.Orlov, T.K.Sedelnikov*, "Atomnaja Energiya", 9, 252 (1957).
10. *A.A.Lukjanov, V.V.Orlov*, "Atomnaya Energija", 10, 262 (1961).
11. *G.I.Marchuck, F.F.Mikhylus*, "Atomnaja Energija", 4, 220 (1958).
12. *A.A.Lukjanov, V.V.Orlov* in "Neutron Physics". M. Gosatomisdat, 1961, p.105.
13. *M.N.Nikolaev, V.V.Filippov, I.I.Bondarenko*, "Atomnaja Energija", II, 445 (1961).
14. *M.N.Nicolaev, V.V.Filippov*, "Atomnaja Energija", 15, 493, (1963).
15. *A.A.Lukjanov*, "Effects of Resonance Structure of Cross-sections of Neutron absorption Slowing down and diffusion in media". Thesis. 1963.
16. *L.P.Abagian, N.O.Bazaziants, I.I.Bondarenko, M.N.Nicolaev*, "Multigroup constants for reactor calculations". M; Gosatomisdat, 1964.
17. *G.L.Marchuck*, "Methods of reactor calculations". Gosatomisdat. M. 1964.
18. *A.M.Baldin, V.I.Goldanski, I.L.Rosental*, "Kinamatics of Nuclear Reactions". M. Phyzmathiz. 1959.
19. *V.N.Faddeeva, M.V.Terentiev*, "Tables of probability-integral values for complex arguments". M. Phyzmatgiz, 1956.
20. *C.Porter, R.Thomas*, Phys. Rev. 104, 483 (1956).
21. *A.Lane, J.Lynn*, Proc. Phys. Soc. A–70, 557 (1957).
22. *R.O.Lane, Jr., A.Langsdorf, J.E.Monahan*, "Annals of Physics", 12, 135 (1961).
23. *G.E.Blokhin* et. al. P N81 in Phys FIR p.399 Vienna 1962.
24. *A.I.Leipunski* et. al. P/2129. Proceedings of Second UN Conference on PUAE. Geneva. 1958.
25. *A.I.Leipunski*, et al. Report N 81 in "Physics of fast and intermediate reactors", p.399, IAEA, Vienna, 1962.
26. *D.A.Broder* et al. P/2177. Proceedings of second UN conference on PUAE. Geneva. 1958.
27. *B.G.Carlson, G.I.Bell* P/2386. Proceedings of Second UN Conference on PUAE. Geneva. 1958.
28. *V.S.Vladimirov* in "Calculational Mathematics" N3, 1958.
29. *H.Bethe* in "Progress in Nuclear Energy". Pergamon Press. 1956.
30. *S.Oleksa*, J. Nucl. Energy 5, 16, 1957.
31. *M.L.Yeater, R.W.Hockenbury, R.R.Fullwood*, Nucl. Sci. and Eng. 2, 105 (1961).
32. *C.Reich, M.Moore* Phys. Rev. 118, 718 (1960).
33. *K.P.Ignatiev, I.V.Kirpichnikov, S.I.Suchoruchkin, "Atomnaja Energija", 16, 110, 1964.*
34. *F.D.Brooks* Proceedings of the Symposium on Neutron time-of-flight methods. Session 2, p.131. Saclay, 1961.
35. *L.N.Usachev, V.A.Pavlinchouck, N.S.Rabotnov* "Atomnaja Energija" in print.
36. *N.O.Bazaziantz, I.V.Gordeev* "Atomnaja Energija" 13, 321, (1962).
37. *P.Greebler, R.A.Hutchins* in "Physics of fast and Intermidiate Reactors" vol 3, P.121. IAEA, Vienna, 1962.
38. *I.V.Gordeev, D.A.Kardashev, A.V.Malishev* "Constans of Nuclear Physics", M.Gosatomisdat, 1963.
39. *G. de Saussure, L.W.Weston* etal. Nucl. Sci. Abstract vol 18, p.151 (1024), 1964.

357

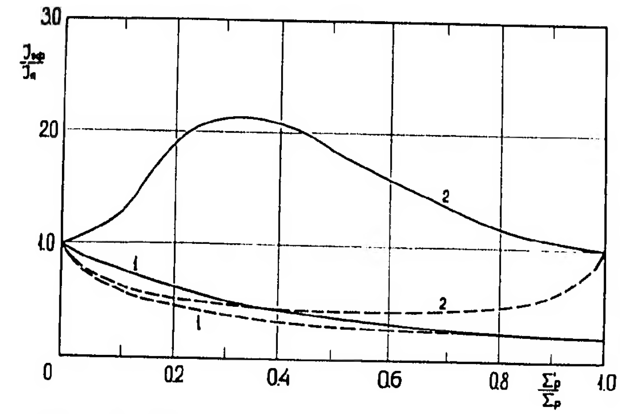


Fig. 1. The dependence of the resonance absorption integral for 29 keV level of Fe^{56} on the dilution of natural Fe by the substance with large energy loss in scattering and cross-sections $\tilde{\Sigma} = \Sigma'_p - \Sigma_p$

Dotted curves were calculated neglecting fluctuations of collision density. The results corresponding to these fluctuations taken into account are shown by solid curves. Interference of resonance and potential scattering was: 1) neglected, 2) considered

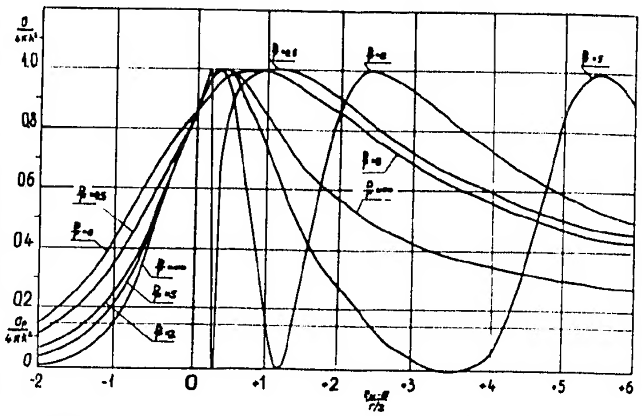
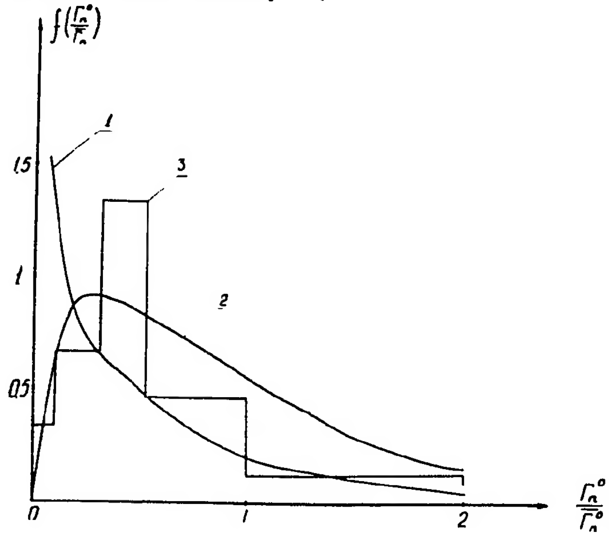


Fig. 2. Cross-section trend in vicinity of two close resonances of even-even nucleus with potential scattering supposed to be determined by S-wave only.



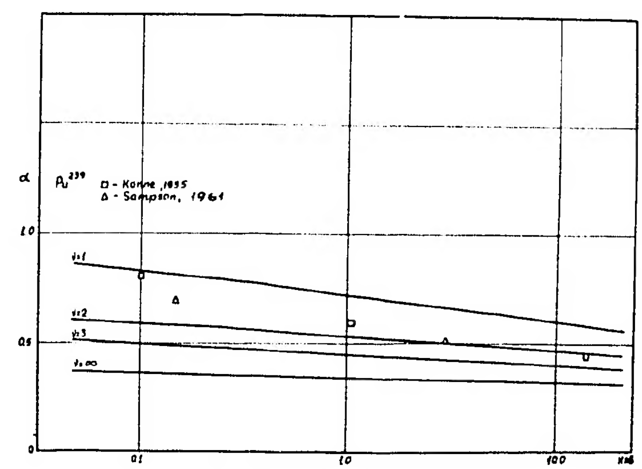


Fig. 3. Calculated and experimental values of α for Pu^{239} are compared. Average resonance parameters are: $\bar{\Gamma}^0_f = 40$ MeV, $\bar{\Gamma}_f = 160$ MeV, $\bar{\Gamma}^0_n = 0.6$ MeV [33], [38]. Experimental distribution corresponds to $\nu = 1$ for both sets of levels [35]. Experimental values of α are in agreement with such assumption within the limits of statistical errors.

Fig. 4. Experimental gistogram (3) of statistical distribution of reduced neutron widths for U^{235} resonances from 11-th to 40-th are compared with one-channel χ^2 - distribution (I) and with distribution calculated in 35 which takes level missing into account